Prekidne Galerkinove metode konačnih elementa: analiza i primene

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Cilj ovog rada je da se razvije teorija konvergencije prekidnih Galerkinovih metoda za aproksimaciju kvazilinearnih eliptičkih i hiperboličkih sistema parcijalnih diferencijalnih jednačina oblika

$$-\sum_{\alpha=1}^{d} \partial_{x_\alpha} S_{i\alpha}(\nabla u(x)) = f_i(x), \qquad i = 1, \ldots, d,$$

i

$$\partial_t^2 u_i - \sum_{\alpha=1}^{d} \partial_{x_\alpha} S_{i\alpha}(\nabla u(t,x)) = f_i(t,x), \qquad i = 1, \ldots, d,$$

gde $\partial_{x_\alpha} := \partial/\partial x_\alpha$, u ograničenoj oblasti $\Omega \subset \mathbb{R}^d$, uz granične uslove mešovitog Dirichlet–Neumannovog tipa i pretpostavljajući da je tenzor $S = (S_{i\alpha})$ uniformno monoton na $\mathbb{R}^{d\times d}$. Pridruženi funkcional energije je tada uniformno konveksan. Dokazaćemo optimalan red konvergencije metode u oba slučaja (tj. u slučaju eliptičkog kao i hiperboličkog zadatka), bez pretpostavke da S zadovoljava globalan Lipschitzov uslov. Nakon toga ćemo oslabiti pretpostavke: koristeći razlomljenu verziju Gårdingove nejednakosti proširićemo rezultat na slučaj kvazilinearnih hiperboličkih sistema jednačina gde se, umesto pretpostavke da je funkcija S uniformno monotona, jedino pretpostavlja da tenzor četvrtog reda $A = \nabla S$ zadovoljava Legendre–Hadamardov uslov. Pridruženi funkcional energije je tada samo rang-1-konveksan. Evolucione jednačine ovakvog tipa pojavljuju se kao matematički modeli u teoriji nelinearne elastičnosti.

Discontinuous Galerkin Finite Element Approximation: Analysis and Applications

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We develop the convergence analysis of discontinuous Galerkin finite element approximations to second-order quasilinear elliptic and hyperbolic systems of partial differential equations of the form, respectively,

$$-\sum_{\alpha=1}^{d} \partial_{x_\alpha} S_{i\alpha}(\nabla u(x)) = f_i(x), \qquad i = 1, \ldots, d,$$

and

$$\partial_t^2 u_i - \sum_{\alpha=1}^{d} \partial_{x_\alpha} S_{i\alpha}(\nabla u(t,x)) = f_i(t,x), \qquad i = 1, \ldots, d,$$

with $\partial_{x_\alpha} = \partial/\partial x_\alpha$, in a bounded spatial domain in $\mathbb{R}^d$, subject to mixed Dirichlet–Neumann boundary conditions, and assuming that $S = (S_{i\alpha})$ is uniformly monotone on $\mathbb{R}^{d\times d}$. The associated energy functional is then uniformly convex. An optimal order bound is derived on the discretization error in each case without requiring the global Lipschitz continuity of the tensor S. We then further relax our hypotheses: using a broken Gårding inequality we extend our optimal error bounds to the case of quasilinear hyperbolic systems where, instead of assuming that S is uniformly monotone, we only require that the fourth-order tensor $A = \nabla S$ is satisfies a Legendre–Hadamard condition. The associated energy functional is then only rank-1 convex. Evolution problems of this kind arise as mathematical models in nonlinear elastic wave propagation.